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OSCILLATIONS OF THE PLASTIC WAVE FRONT UNDER HIGH-RATE LOADING

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Two models of elastoplastic wave propagation in metals under uniaxial deformation are considered. The first model treats plastic deformation as being due to dislocation motion during heterogeneous formation of dislocations. The second model assumes that plastic deformation occurs by motion of dipoles of partial disclinations. It is shown that in both cases, certain conditions can give rise to damped oscillations of the plastic wave front, which were detected in shock loading experiments with flat specimens made of 28Kh3SNMFA *steel.*

The elastoplastic deformation of metals under shock-wave loading and rapid changes of their defect structure at the microlevel during shock-wave propagation should be reflected adequately in physical and mathematical models of material's behavior. Deformation hardening, relaxation of shear strains, diffusion of rarefaction waves due to the Baushinger effect, etc., are described primarily by dislocation models [1, 2] and more rarely by disclination [3] models of relaxation type.

However, a number of experiments on high-rate loading of materials revealed damped oscillations of the plastic wave front, which are not described by existing models. The procedure of these experiments, based on recording the motion of the free surface of shock-loaded specimens (disks of 52 mm diameter and 10 mm thickness) by laser interferometry, is described in detail in a number of papers (see, e.g., [4]).

Damped oscillations were recorded for 28Kh3SNMFA steel under initial impact velocities of $V_0 = 80$, 84, 97.5, 100, 100.6, 130.9, 142.2, 150, 311, 376, and 384 m/sec.

A typical curve of the oscillations is shown in an interferogram (Fig. 1) and an U–t diagram (Fig. 2) for a sample loaded at an impact velocity of 311 m/sec.

In all cases, the oscillations at the plastic wave front last for not more 0.15–0.3 μsec and arise immediately after passage of an elastic precursor, which suggests significant structural changes in the metal after passage of the precursor.

Taking into account modern concepts of the physics of strength and plasticity [5–8], one might expect that during dynamic loading in this case (with allowance for the degree of alloying of the steel) the heterogeneous mechanism of dislocation formation is most probable. This mechanism can work both on the elastic precursor and at the plastic wave front. In addition, if behind the elastic precursor, the dislocation density reaches a critical value, then some of the dislocations, which were previously distributed chaotically, gather (near those available in the material) to form finite walls, which, being nuclei of rotational deformation, i.e., dipoles of partial disclinations, form boundaries of cells (fragments) [7, 8]. Because interaction of dipoles initiates the production of new dipoles by the already available dipoles of partial disclinations [8], the process of change in internal structure is similar to the heterogeneous mechanism of dislocation formation. Furthermore, during the structural change, plastic deformation occurs only by motion of the dipoles.

Let us consider the first version of development of the process with plastic deformation occurring by dislocation motion during heterogeneous dislocation formation and derive the constitutive equation describing elastoplastic wave propagation under uniaxial deformation.

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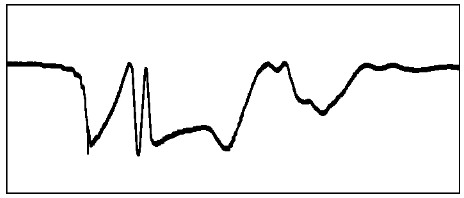

Fig. 1

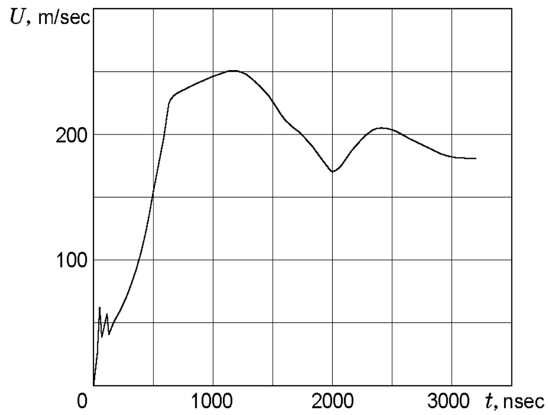

Fig. 2

A possible mechanism of avalanche generation of dislocations is the so-called kinematic mechanism [9]. This mechanism assumes that because of the instability of the core, every dislocation moving at a high velocity generates two new dislocations, one of which continue moving together with the primary dislocation, and the second, opposite in sign, moves backward. In this mechanism, dislocation multiplication occurs in an avalanche manner, and mobile dislocations generated by cross slip under the regenerative law can be primary.

For a quantitative consideration of dislocation generation by the mechanism described above, we denote by f the ratio of the density heterogeneously generated dislocations to the density of mobile dislocations multiplied by the regenerative mechanism:

$$f = N_{mr}/(N_{m0} + \alpha\gamma), \tag{1}$$

where N_{m0} is the initial density of mobile dislocations, α is the dislocation multiplication factor, and γ is the shear strain. With excess of a certain threshold amplitude of the pulsed pressure, the increment of the quantity f (1) increases in proportion to the shear strain rate [10]:

$$df \sim d\gamma. \tag{2}$$

Obviously, the increment (2) of relation (1) should increase in proportion to this ratio f because new dislocations act as new sources:

$$df = kf\, d\gamma, \tag{3}$$

where k is the proportionality factor which characterizes the rate of heterogeneous formation of dislocations. Since heterogeneous multiplication, in contrast to homogeneous multiplication, implies the material contain strain concentrators, which produce significant inhomogeneity of the stress field, the factor k should depend on the stress applied. It is known that with increase in shear stress, the efficiency of stress concentrators reduces, and, therefore, one might expect that with increase in shear stress, the coefficient k decreases:

$$k = H_1/\tau. \tag{4}$$

Here H_1 is a constant and τ is the shear stress.

Integration of Eq. (3) over time taking into account (1) and (4) yields the following dependence for the density of heterogeneously generated mobile dislocations:

$$N_{mr} = N_{mp} \exp\,(H_1\dot{\gamma}/\tau).$$

Here N_{mp} is the density of homogeneously generated dislocations; the dot above the symbol denotes differentiation with respect to time along the pathway of an element of the medium.

The total density of mobile dislocations due to both multiplication mechanisms is given by

$$N_m = N_{mp} + N_{mr} = N_{mp}(1 + \exp(H_1\dot{\gamma}/\tau)). \tag{5}$$

Since heterogeneous formation of dislocations proceeds in an avalanche manner and at high strain rates, the unity in (5) can be ignored. As a result, we obtain

$$N_m = (N_{m0} + \alpha\gamma)\exp(H_1\dot{\gamma}/\tau). \tag{6}$$

Besides relation (6), which defines the density of mobile dislocations, we need to obtain the relation between the dislocation velocity V and the shear strain rate. Many experimental data are well fitted by the exponential dependence

$$V = c_t \exp((-\tau_0 + H\gamma)/\tau), \tag{7}$$

where c_t is the velocity of transverse sound waves, τ_0 is the characteristic stress of dislocation deceleration, and H is the hardening constant of the material, which takes into account the stopping of dislocations by each other at their high density.

An increase in the mobility of dislocations with increase in strain rate can be allowed for by reducing the characteristic stress τ_0 in a dependence similar to (7). As a result, the dependence of dislocation velocity on strain and strain rate can be written as

$$V = c_t \exp((-\tau_0 + H_2\dot{\gamma})/\tau), \tag{8}$$

where H_2 is a proportionality factor. Expression (8) implies that the mobility of dislocations is of a dual nature, i.e., it is due to thermal activation and athermic processes. The term τ_0, whose temperature dependence was studied in [11, 12], allows for thermal activation processes, and the term $H_2\dot{\gamma}$ allows for athermic processes. Substituting (6) and (8) into the well-known equation for the plastic shear strain rate $\dot{\gamma} = bN_mV$ (b is the Burgers vector), and then into the equation for the uniaxial strain state in an elastoplastic material behind the plane shock wave front:

$$\sigma - (\lambda + 2G)\varepsilon = -(8/3)G\gamma.$$

Here λ and G are Lamé's parameters, σ is the normal stress in the direction of wave propagation, and ε is the total (elastic and plastic) strain in the wave propagation direction, we obtain the constitutive equation

$$\sigma - \rho_0 c^2\varepsilon = -(8/3)Gbc_t(N_m + \alpha\gamma)\exp((-\tau_0 - (H_1 + H_2)\dot{\gamma})/\tau), \tag{9}$$

where c is the longitudinal velocity of sound.

Because an increase in the density of mobile dislocations [see (6)] and an increase in their velocity [see (8)] lead to an increase in total strain rate and are indistinguishable at the macroscopic level, the sum of the coefficients H_1 and H_2 in Eq. (9) can be replaced by one coefficient.

Using the relations of uniaxial deformation of an isotropic material

$$\sigma - (\lambda + 2G)\varepsilon = -(8/3)G\gamma, \tag{10}$$

$$\tau = (3/4)(\sigma - (\lambda + (2/3)G)\varepsilon), \tag{11}$$

we can write Eq. (9) in terms of shear and total strain only:

$$\dot{\gamma} = \gamma_*(1 + M\gamma)\exp((-\tau_* - B_*\dot{\gamma})/(\varepsilon - 2\gamma)). \tag{12}$$

Here $\gamma_* = bc_tN_{m0}$, $M = \alpha/N_{m0}$, $\tau_* = \tau_0/G$, and $B_* = (H_1 + H_2)/G$.

For uniaxial deformation of an isotropic material, the constitute equation (12) combined with the continuum dynamics equations

$$\rho_0 \frac{\partial u_1}{\partial t} + \frac{\partial \sigma_{11}}{\partial x} = 0, \qquad \frac{\partial u_1}{\partial x} + \frac{\partial \varepsilon_{11}}{\partial t} = 0 \tag{13}$$

forms a closed system of equations which describe dynamic deformation with allowance for inertial effects. Here u_1 is the rate of motion of material particles in the wave propagation direction x, σ_{11} is the strain component in the wave propagation direction, and ε_{11} is the total (elastic and plastic) strain component in the wave propagation direction. Below, the subscript 11 at the stress and strain components is omitted, and the subscripts t and x denote differentiation with respect to time and coordinate, respectively. Equations (13) can be reduced to the second-order equation $\sigma_{xx} - \rho_0\varepsilon_{tt} = 0$, which, using relation (10), can also be expressed in terms of total and shear strains:

$$\varepsilon_{xx} - (1/c^2)\varepsilon_{tt} - (8/3)(G/(\rho_0 c^2))\gamma_{xx} = 0. \tag{14}$$

From Eq. (12) we express the total strain:

$$\varepsilon = 2\gamma - \tau_*/\delta + (B_*\gamma_t)\delta. \tag{15}$$

Here $\delta = \ln(\gamma_t/(\gamma_*(1 + M\gamma)))$, and, substituting (15) into (14), we obtain the equation

$$\begin{aligned}\delta(\tau_* - B_*\gamma_t)(\delta_{xx} - (1/c^2)\delta_{tt}) - 2B_*\delta(\gamma_x\delta_x - (1/c^2)\gamma_t\delta_t) - \delta^2 B_*(\gamma_{txx} - (1/c^2)\gamma_{ttt})\\ - 2(\tau_* - B_*\gamma_t)(\delta_x^2 - (1/c^2)\delta_t) + \delta^3((2 - (8/3)G/(\rho_0 c^2))\gamma_{xx} - (1/c^2)\gamma_{tt}) = 0.\end{aligned} \tag{16}$$

The solution of Eq. (16) is sought in exponential form

$$1 + M\gamma = \Gamma(x,t)\exp(-K(x - c_p t)), \tag{17}$$

where the preexponent $\Gamma(x,t)$ is generally a function of the coordinates and time, $c_p = \omega/K$ is the velocity of the stationary plastic front, and K is the wave number.

In the particular case of $\Gamma = \text{const}$, δ is also a constant and Eq. (17) is linearized:

$$(2 - (8/3)G/(\rho_0 c^2))\gamma_{xx} - (2/c^2)\gamma_{tt} - (B_*/\delta)(\gamma_{txx} - (1/c^2)\gamma_{ttt}) = 0. \tag{18}$$

Here $\delta = \ln(\omega/(\gamma_* M)) = \text{const}$.

The partial equation (18) corresponds to the dispersion equation

$$B_*\omega/z = \ln(\omega/(\gamma_* M)), \tag{19}$$

where $z = 2(c_r^2 - c_p^2)/(c^2 - c_p^2)$ (c_r is the volumetric velocity of sound in the medium). In Eq. (19), we expand the exponent in a series, restricting ourselves to quadratic terms of the expansion. We obtain

$$\omega/(\gamma_* M) = 1 + B_*\omega/z + (1/2)B_*^2\omega^2/z^2,$$

whence

$$\omega = \frac{z^2}{B_*^2}\Big(\frac{1}{\gamma_* M} - \frac{B_*}{z} \pm \Big(\frac{z^2}{B_*^2}\Big(\frac{1}{\gamma_* M} - \frac{B_*}{z}\Big) - 2\Big)^{1/2}\Big). \tag{20}$$

From expression (20) it follows that the quantity ω is real for $(z^2/B_*^2)(1/(\gamma_* M) - B_*/z)^2 > 2$ and complex for $(z^2/B_*^2)(1/(\gamma_* M) - B_*/z) < 2$. The condition of existence of complex roots implies that the plastic front has an oscillation structure, and this condition is satisfied for $((z/B_*)(1/(\gamma_* M) - B_*/z) - 2^{1/2})((z/B_*)(1/(\gamma_* M) - B_*/z) + 2^{1/2}) < 0$ in one of the two cases:

$$\begin{aligned}&(z/B_*)(1/(\gamma_* M) - B_*/z) - 2^{1/2} < 0, \qquad (z/B_*)(1/(\gamma_* M) - B_*/z) + 2^{1/2} > 0,\\ &(z/B_*)(1/(\gamma_* M) - B_*/z) - 2^{1/2} > 0, \qquad (z/B_*)(1/(\gamma_* M) - B_*/z) + 2^{1/2} < 0.\end{aligned} \tag{21}$$

Inequalities (21) are equivalent to the following two conditions imposed on the wave propagation velocity:

$$c^2/(1 - 2/(B_*\gamma_* M(1 + 2^{1/2}))) < c_p^2 < c^2/(1 - 2/(B_*\gamma_* M(1 - 2^{1/2})));$$

$$c^2/(1 - 2/(B_*\gamma_* M(1 - 2^{1/2}))) < c_p^2 < c^2/(1 - 2/(B_*\gamma_* M(1 + 2^{1/2}))). \tag{22}$$

However, in the elastoplastic flow region, only inequality (22), which defines values of the dislocation parameters α, b, H_1, and H_2 for which the occurrence of oscillations of the plastic wave front is possible.

Let us consider the second version of development of the process in which plastic deformation occurs by formation and motion of dislocation walls, i.e., by motion of dipoles of partial disclinations. In this case, the rate of plastic strain due to motion of dipoles is defined by the equation

$$\dot{\varepsilon} \simeq 2n\omega_1 a V_{\rm d}, \tag{23}$$

where n, $V_{\rm d}$, $2a$, and ω_1 are the density, mean velocity, reach of arm, and power (Frank vector) of the dipoles.

The density of the dipoles n is determined from the equation for the density of the walls at the moment of their heterogeneous generation [8]

$$\frac{dn}{dt} = \alpha_1 N_m^2 - \beta N_m n. \tag{24}$$

Here N_m is the density of mobile dislocations, α_1 is the probability of formation of a "nuclei" of a wall during meeting of two dislocations and its growth up to formation of a complete wall, and β is the probability of collapse of the wall under the action of an oncoming dislocation.

The velocity of the dipole is determined from the dependence [6]

$$V_{\rm d} = -\int_{-a}^{a} \frac{V(y)}{\lambda_1 \ln(1 - \omega_1/(bN_1\lambda_1))}\, dy, \tag{25}$$

where $V(y)$ is the dislocation velocity in the zone of rotational plasticity $y_1 \leqslant y \leqslant y_2$, y_1 and y_2 are generally functions of external stress and the reach of arm a and power of the dipole ω_1 (below, we set $y_1 = -a$ and $y_2 = a$), λ_1 is the mean free path of dislocations in the zone of rotational plasticity, and N_1 is the initial dislocation density in the zone of rotational plasticity.

Assuming that the function $V(y)$ does not depend on the coordinate y and that $\lambda_1 \simeq a$, from Eq. (25) we obtain

$$V_{\rm d} = -V/f_1, \tag{26}$$

where $f_1 = \ln(1 - \omega/(bN_1 a))$.

Solving Eq. (24) for n and substituting the obtained solution together with the value of $V_{\rm d}$ from relation (26) into Eq. (23) and taking into account that $\dot\varepsilon = -(\ddot u/c_p)$, we obtain

$$\ddot u = (2\omega_1 a V \alpha_1 N_m(\exp(\beta N_m t) - 1) + \beta n_0)c_p/(f_1 \beta \exp(\beta N_m t)), \tag{27}$$

where n_0 is the initial density of dislocation walls or dipoles.

For further transformation of Eq. (27), we use the approximate equality $\exp(\beta N_m t) \simeq 1 + \beta N_m t$. In addition, in the denominator of the equation, we set $\exp(\beta N_m t) \simeq 1$ (this assumption is confirmed by numerical calculation). Then, taking into account that $t = u/\dot u$, from Eq. (27) we obtain the equation

$$\ddot u = (2N_m^2 V c_p \alpha_1 \beta \omega_1 a u + \beta n_0 c_p u)/(f_1 \beta \dot u). \tag{28}$$

From the calculations it follows that in the examined range of shock velocities, $f_1 \dot u \simeq -1$ m/sec, and, hence, (28) becomes the equation

$$\ddot u + 2h\dot u + \omega_0 u = 0, \tag{29}$$

where $2h = n_0 c_p$ and $\omega_0 = 2N_m V c_p \alpha_1 \omega_1 a$.

As is known, if $h^2 < \omega_0^2$, Eq. (29) defines an oscillatory process. In our case, this condition is satisfied for

$$2N_m V \alpha_1 \omega_1 a > n_0/2, \tag{30}$$

i.e., inequality (30) gives values of the parameters of the dislocation–disclination structure (N_m, V, α_1, ω_1, a, and n_0) for which the occurrence of oscillations of plastic flow is also possible.

Using parameter values typical of the processes considered, we estimate the probability of formation of a wall nucleus α_1. Let $N_m = 10^{12}$ m^{-2}, $\omega_1 = 6 \cdot 10^{-3}$, $2a = 0.2 \cdot 10^{-6}$ m, $V = 10^2$ m/sec, and $n_0 = 10^7$–10^8 [6]. Then, condition (30) is satisfied for $\alpha_1 > 10^{-9}$–10^{-8}.

It should be noted that the role of the dissipative term in Eq. (29) is played by dislocation walls that were available in the material initially or formed during heterogeneous multiplication of dislocations (during motion of the elastic precursor). In the second version of development of internal processes in the material, oscillations of the plastic wave front cease after the cellular structure has completely formed or after the walls, having reached the critical density ($n_* \simeq 10^{11}$m^{-2} [13]), collapse into separate dislocations. In the first case, both the constitutive equations of dislocation plasticity [14] and the conditions of dislocation motion and multiplication change.

The second statement is easy to verify. For this, we write the solution of Eq. (24):

$$N = (\alpha_1 N_m(\exp(\beta N_m t) - 1) + \beta n_0)/(\beta \exp(\beta N_m t)). \tag{31}$$

Setting $\alpha_1 = \beta = 10^{-7}$, $N_m = 10^{12}$ m^{-2}, and $n_0 = 10^7$ m^{-2} and substituting these values and $t = 0.15$ μsec into relation (31), we obtain the value $n = 1.9 \cdot 10^{11}$ m^{-2}, which has the same order of magnitude as n_*. This confirms that the assumptions adopted in constructing the model of occurrence of damped oscillations of the plastic wave front are valid.

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